

Inequivalence Between Passive Gravitational Mass and Energy for a Quantum Body: Theory and Suggested Experiment

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Recently, we have suggested some semi-quantitative Hamiltonian for an electron in a hydrogen atom in a weak gravitational field, which takes into account quantum effects of electron motion in the atom. We have shown that this Hamiltonian predicts breakdown of the equivalence between passive electron gravitational mass and its energy. Moreover, as has been shown by us, the latter phenomenon can be experimentally observed as unusual emission of radiation from an ensemble of the atoms, provided that they are moved in the Earth's gravitational field with constant velocity by some spacecraft. In this article, we derive the above-mentioned Hamiltonian from the Dirac equation in a curved spacetime. It is shown that it exactly coincides with the semi-quantitative Hamiltonian, used in our previous papers. We extend the obtained Hamiltonian to the case of a spacecraft, containing a macroscopic ensemble of the atoms and moving with a constant velocity in the Earth's gravitational field. On this basis, we discuss some idealized and realistic experiments on the Earth's orbit. If such (realistic) experiment is done, it will be the first direct observation of quantum effects in the General Relativity.

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1. INTRODUCTION

It is well known that creation of the Quantum Gravitation theory is the most important step in developing of the so-called Theory of Everything. This problem appears to be extremely difficult. This is partially due to the fact that the foundations of quantum mechanics and the General Relativity are very different and partially due to the absence of the corresponding experimental data. So far, quantum effects have been directly observed only in the Newtonian variant of gravitation [1, 2]. On the other hand, such important quantum phenomenon in the General Relativity as the Hawking radiation [3] is still very far from its direct experimental discovery. In this complicated situation, we have suggested two novel phenomena [4–8], which show that passive and active gravitational masses of a composite quantum body are not equivalent to its energy due to some quantum effects. Moreover, we have also proposed experimental ways [4–8] how to observe the above mentioned phenomena. If one of these experiments is done, it will be the first direct observation of quantum effects in the General Relativity. In this paper, we concentrate our consideration on passive gravitational mass of the simplest composite quantum body - a hydrogen atom. The main our results are derivations of two Hamiltonians from the Dirac equation in curved spacetime of the General Relativity, which were semi-qualitatively introduced by us in Refs.[4–8]. Note that a notion of passive gravitational mass of a composite body is not trivial even in classical physics. As mentioned by Nordtvedt [9] and Carlip [10], an external weak gravitational field is coupled with the combination $m_e + m_p + (3K + 2P)/c^2$, where c is the velocity of light, m_e and m_p are the bare masses of electron and proton, K and P are kinetic and potential energies of an electron. Nevertheless, due to the classical virial theorem averaged over time combination $\langle 2K + P \rangle_t = 0$ and, therefore, averaged over time electron gravitational mass, $\langle m_g \rangle_t$, satisfy the famous Einstein's equation,

$$\langle m_g \rangle_t = m_e + \left\langle \frac{3K + 2P}{c^2} \right\rangle_t = m_e + \left\langle \frac{K + P}{c^2} \right\rangle_t = \frac{E}{c^2}, \quad (1)$$

where E is the total electron energy. On this basis, in Refs.[9, 10], the conclusion about the equivalence between averaged over time passive gravitational mass and energy was made. Recently, in Refs.[4–8], we have considered a quantum hydrogen atom in a weak gravitational field. We have suggested semi-quantitative derivation of its post-Newtonian Hamiltonian, ignoring the so-called tidal corrections, magnetic force, and all spin related phenomena. As a result, we have made the following three statements about passive gravitational mass [4–8]: (a) The expectation value of the mass for a stationary electron quantum state is equivalent to its energy; (b) For an individual measurement, there exists small probability that the mass is not equivalent to the energy; (c) The above mentioned inequivalence of the mass and energy can be experimentally observed as a detection of unusual radiation, emitted by a macroscopic ensemble of the atoms, moved with constant velocity by spacecraft in the Earth's gravitational field.

2. SOME OF OUR PREVIOUS RESULTS [4–8]

In Subsection 2.1, we describe in brief semi-quantitative derivation of the quantum Hamiltonian for a hydrogen atom in a weak gravitational field. In Subsection 2.2, we discuss how the quantum virial theorem [11] results in a surviving of the equivalence between the expectation value of electron passive gravitational mass and its energy. In Subsection 2.3, we show that the equivalence between electron passive gravitational mass and its energy is broken with a small probability during individual measurements of the mass.

2.1. Semi-Quantitative Derivation of the Hamiltonian [4–8]

Let us write a standard interval, describing spacetime in a weak field approximation outside some massive body [12],

$$ds^2 = -\left(1 + 2\frac{\phi}{c^2}\right)(cdt)^2 + \left(1 - 2\frac{\phi}{c^2}\right)(dx^2 + dy^2 + dz^2), \quad \phi = -\frac{GM}{R}, \quad (2)$$

where G is the gravitational constant, M is the body mass, and R is the distance between its center and center of mass of a hydrogen atom. Then, according to the General Relativity we can introduce the following local proper spacetime coordinates,

$$x' = \left(1 - \frac{\phi}{c^2}\right)x, \quad y' = \left(1 - \frac{\phi}{c^2}\right)y, \quad z' = \left(1 - \frac{\phi}{c^2}\right)z, \quad t' = \left(1 + \frac{\phi}{c^2}\right)t, \quad (3)$$

where the Schrödinger equation for electron in a hydrogen atom can be approximately expressed in its standard form,

$$i\hbar \frac{\partial \Psi(\mathbf{r}', t')}{\partial t'} = \hat{H}_0(\hat{\mathbf{p}}', \mathbf{r}') \Psi(\mathbf{r}', t'), \quad (4)$$

where

$$\hat{H}_0(\hat{\mathbf{p}}', \mathbf{r}') = m_e c^2 - \frac{\hat{\mathbf{p}}'^2}{2m_e} - \frac{e^2}{r'}. \quad (5)$$

We point out that Eqs.(2)-(5) are written in the so-called $1/c^2$ approximation. As to gravitation, we take into account only terms of the order of ϕ/c^2 , which can be estimated as 10^{-9} near the Earth, and disregard terms of the order of $(\phi/c^2)^2 \sim 10^{-18}$. We also stress that, in Eqs.(4),(5), we do not take into account the so-called tidal effects (i.e., we do not differentiate gravitational potential ϕ with respect to electron coordinates, \mathbf{r} and \mathbf{r}'). Note that \mathbf{r} and \mathbf{r}' correspond to electron positions in the center of mass coordinate system, which we relate to proton position. In Section 3, we show that, in fact, this means that we consider a hydrogen atom as a point-like body and disregard all tidal terms in the electron Hamiltonian, which are usually very small and are of the order of $(r_B/R_0)|\phi/c^2| \sim 10^{-17}|\phi/c^2| \sim 10^{-26}$ in the Earth's gravitational field. Here, r_B is the so-called Bohr's radius and R_0 is the Earth's radius. In Eqs.(4),(5), we also disregard magnetic force and all spin related effects. Another our suggestion is that proton mass is very high, $m_p \gg m_e$, and, thus, proton can be considered as a classical particle, whose position is fixed and kinetic energy is negligible. In this article, as usual, we consider the weak gravitational field (2) as a perturbation in some inertial coordinate system. The inertial coordinate system corresponds to Minkowskii spacetime coordinates (x, y, z, t) in Eq.(3), where it is possible to obtain the following electron Hamiltonian from Eqs.(3)-(5):

$$\hat{H}(\hat{\mathbf{p}}, \mathbf{r}) = m_e c^2 + \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} + m_e \phi + \left(3\frac{\hat{\mathbf{p}}^2}{2m_e} - 2\frac{e^2}{r}\right)\frac{\phi}{c^2}. \quad (6)$$

Note that the Hamiltonian (6) can be rewritten in more convenient form,

$$\hat{H}(\hat{\mathbf{p}}, \mathbf{r}) = m_e c^2 + \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} + \hat{m}_g(\hat{\mathbf{p}}, \mathbf{r})\phi, \quad (7)$$

where we introduce the following electron passive gravitational mass operator:

$$\hat{m}_g(\hat{\mathbf{p}}, \mathbf{r}) = m_e + \left(\frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r}\right)\frac{1}{c^2} + \left(2\frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r}\right)\frac{1}{c^2}, \quad (8)$$

which is proportional to electron weight operator in the weak gravitational field (2). It is important that, in Eq.(8), the first term corresponds to the bare electron mass, m_e , the second term corresponds to the expected electron energy contribution to the mass operator, whereas the third term is the non-trivial virial contribution to passive gravitational mass operator.

2.2. Equivalence Between the Expectation Value of Mass and Energy [4–8]

In this Subsection, we discuss one important consequence of Eqs.(7),(8). We stress that the operator (8) does not commute with taken in the absence of the gravitational field electron energy operator. Therefore, from the beginning, it seems that there is no any equivalence between electron passive gravitational mass and its energy. To establish their equivalence at a macroscopic level, we consider a macroscopic ensemble of hydrogen atoms with each of them being in a stationary ground state with energy E_1 ,

$$\Psi_1(r, t) = \Psi_1(r) \exp\left(\frac{-im_e c^2 t}{\hbar}\right) \exp\left(\frac{-iE_1 t}{\hbar}\right), \quad (9)$$

where $\Psi_1(r)$ is a normalized electron ground state wave function in a hydrogen atom. As follows from Eq.(8), in this case, the expectation value of passive gravitational mass operator per one electron is

$$\langle \hat{m}_g \rangle = m_e + \frac{E_1}{c^2} + \left\langle 2 \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} \right\rangle \frac{1}{c^2} = m_e + \frac{E_1}{c^2}, \quad (10)$$

where the expectation value of the virial term in Eq.(10) is zero due to the quantum virial theorem [11]. Note that the result (10) can be easily extended to any stationary quantum state in a hydrogen atom. Thus, we can conclude that the equivalence between passive gravitational mass and energy survives at a macroscopic level for stationary quantum states.

2.3. Breakdown of the Equivalence Between Passive Gravitational Mass and Energy at a Microscopic Level [4–8]

Here, we describe a thought experiment, which directly demonstrates inequivalence between passive gravitational mass and energy at a microscopic level. Let us consider electron with a ground state wave function (9) in a hydrogen atom, corresponding to the absence of the gravitational field (2),

$$\hat{H}_0(\mathbf{p}, r) \Psi_1(r) = E_1 \Psi_1(r), \quad \hat{H}_0(\mathbf{p}, r) = m_e c^2 + \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r}. \quad (11)$$

Now, we take into account the gravitational field (2), as a perturbation to the Hamiltonian (11),

$$\hat{H}(\hat{\mathbf{p}}, \mathbf{r}) = \hat{H}_0(\mathbf{p}, r) + \hat{m}_g(\hat{\mathbf{p}}, \mathbf{r}) \phi, \quad (12)$$

where electron passive gravitational mass operator is given by Eq.(8). Let us apply to the Hamiltonian (12) and its ground state wave function, $\tilde{\Psi}_1(r)$, where

$$\hat{H}(\hat{\mathbf{p}}, \mathbf{r}) \tilde{\Psi}_1(r) = \tilde{E}_1 \tilde{\Psi}_1(r), \quad (13)$$

$$\tilde{\Psi}_1(r) = \sum_n a_n \Psi_n(r), \quad (14)$$

the standard quantum mechanical perturbation theory. Note that, in Eq.(14), $\Psi_n(r)$ are normalized electron wave functions in a hydrogen atom in the absence of the gravitation (2), corresponding only to atomic levels nS with energy E_n , due to a special selection rule of the perturbation (8). It is possible to show that, in accordance with the perturbation theory, coefficient a_1 and correction to energy of the ground state can be expressed as:

$$a_1 \simeq 1, \quad \tilde{E}_1 = \left[1 + \frac{\phi(R)}{c^2} \right] E_1, \quad (15)$$

where the last term in Eq.(15) corresponds to the famous red shift in a gravitational field. It manifests the expected contribution to passive gravitational mass due to electron binding energy in a hydrogen atom. Note that during derivation of Eq.(15), we have used the quantum virial theorem [11], which states that

$$\int \Psi_1^*(r) \left(2 \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} \right) \Psi_1(r) d^3\mathbf{r} = 0. \quad (16)$$

As to the coefficients a_n with $n \neq 1$ in Eq.(14), they can be expressed as functions of the virial operator matrix elements,

$$a_n = \left[\frac{\phi(R)}{c^2} \right] \left(\frac{V_{n,1}}{E_1 - E_2} \right), \quad V_{n,1} = \int \Psi_n^*(r) \left(2 \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} \right) \Psi_1(r) d^3\mathbf{r}. \quad (17)$$

Here, we point out that the wave function (14)-(17), corresponding to electron ground state in the presence of the gravitational field (2), is a series of eigenfunctions of electron energy operator in the absence of the field. Therefore, if we measure energy in electron quantum state (14)-(17) by means of operator (11), we obtain the following quantized values:

$$E(n) = m_e c^2 + E_n, \quad (18)$$

where we disregard the red shift effect. Other words, we can conclude that with probability close to 1 [see Eq.(15)], the Einstein equation, $E = m_e c^2 + E_1$, survives in our case, whereas with small probabilities,

$$P_n = |a_n|^2 = \left[\frac{\phi(R)}{c^2} \right]^2 \frac{V_{n,1}^2}{(E_n - E_1)^2}, \quad n \neq 1, \quad (19)$$

it is broken. The reason for this breakdown is that, as follows from Eqs.(14)-(17), electron wave function with definite passive gravitational mass is not characterized by definite energy in the absence of the gravitational field. Below, we would like to discuss two points, related to the fact that the probabilities (19) are of the second order magnitude with respect to small parameter (ϕ/c^2) . First point - the accuracy of our calculations. According to quantum mechanics, it is enough to calculate wave function (14) in a linear approximation with respect to small parameter ϕ/c^2 to obtain probabilities (19). Second point is related to physical meaning of our results. In this article, we do not show that the average passive gravitational mass of electron is not equal to its energy, since we do not take into account terms of the order of $(\phi/c^2)^2$ in Eqs.(2),(3) as well as we do not calculate correction of the order of $(\phi/c^2)^2$ to the ground state of a hydrogen atom. Here, it is important that quantum mechanics, as known, is not a science about average values, but it is a science about probabilities of individual measurements. What we really show is that electron with definite gravitational mass (8) can be excited with small probabilities (19) to high energy levels (18) during quantum measurement of its energy by means of operator (11). Why is that so important? The answer is the following: high energy levels are quasi-stationary and we can expect photons emission from a macroscopic ensemble of the atoms. For description of the corresponding experiment, see Section 4.

3. RESULTS

In this Section, we present in detail the main results of the current article - careful derivations of two Hamiltonians from the Dirac equation in a curved spacetime of the General Relativity. In Subsection 3.1, we present and comment the Hamiltonian for a hydrogen atom, derived in Ref.[13], where the so-called tidal terms are taken into account as well as motion of a center of mass of the atom. In Subsection 3.2, we derive from the Hamiltonian [13] the considered in the previous Section Hamiltonian (7),(8), whereas, in Subsection 3.3, we derive the Hamiltonian, which is used later for description of the suggested experiment (see Section 4).

3.1. The Most General Hamiltonian [13]

In Ref.[13], the mixing effect between even and odd wave functions in a hydrogen atom (i.e., the so-called relativistic Stark effect) was studied in an external gravitational field. A possibility of a center of mass motion of the atom was taken into account and the corresponding Hamiltonian was derived in $1/c^2$ approximation. The peculiarity of the calculations in the above mention article was that not only terms of the order of ϕ/c^2 were calculated, as in our

case, but also terms of the order of ϕ'/c^2 , where ϕ' is a symbolic derivative of ϕ with respect to reciprocal electron coordinates in the atom. According to the existing tradition, we call the latter terms tidal ones. The Hamiltonian (3.24), obtained in Ref. [13] for the corresponding Schrödinger equation, can be written as a sum of the following four terms:

$$\hat{H}(\hat{\mathbf{P}}, \hat{\mathbf{p}}, \tilde{\mathbf{R}}, r) = \hat{H}_0(\hat{\mathbf{P}}, \hat{\mathbf{p}}, r) + \hat{H}_1(\hat{\mathbf{P}}, \hat{\mathbf{p}}, \tilde{\mathbf{R}}, r) + \hat{H}_2(\hat{\mathbf{p}}, \mathbf{r}) + \hat{H}_3(\hat{\mathbf{P}}, \hat{\mathbf{p}}, \tilde{\mathbf{R}}, r), \quad (20)$$

where

$$\hat{H}_0(\hat{\mathbf{P}}, \hat{\mathbf{p}}, r) = m_e c^2 + m_p c^2 + \left[\frac{\hat{\mathbf{P}}^2}{2(m_e + m_p)} + \frac{\hat{\mathbf{p}}^2}{2\mu} \right] - \frac{e^2}{r}, \quad (21)$$

$$\hat{H}_1(\hat{\mathbf{P}}, \hat{\mathbf{p}}, \tilde{\mathbf{R}}, r) = \left\{ m_e c^2 + m_p c^2 + \left[3 \frac{\hat{\mathbf{P}}^2}{2(m_e + m_p)} + 3 \frac{\hat{\mathbf{p}}^2}{2\mu} - 2 \frac{e^2}{r} \right] \right\} \left(\frac{\phi - \mathbf{g} \tilde{\mathbf{R}}}{c^2} \right), \quad (22)$$

$$\hat{H}_2(\hat{\mathbf{p}}, \mathbf{r}) = \frac{1}{c^2} \left(\frac{1}{m_e} - \frac{1}{m_p} \right) [-(\mathbf{g} \mathbf{r}) \hat{\mathbf{p}}^2 + i \hbar \mathbf{g} \hat{\mathbf{p}}] + \frac{1}{c^2} \mathbf{g} \left(\frac{\hat{\mathbf{s}}_e}{m_e} - \frac{\hat{\mathbf{s}}_p}{m_p} \right) \times \hat{\mathbf{p}} + \frac{e^2(m_p - m_e)}{2(m_e + m_p)c^2} \frac{\mathbf{g} \mathbf{r}}{r}, \quad (23)$$

$$\hat{H}_3(\hat{\mathbf{P}}, \hat{\mathbf{p}}, \tilde{\mathbf{R}}, r) = \frac{3}{2} \frac{i \hbar \mathbf{g} \mathbf{P}}{(m_e + m_p)c^2} + \frac{3}{2} \frac{\mathbf{g}(\mathbf{s}_e + \mathbf{s}_p) \times \mathbf{P}}{(m_e + m_p)c^2} - \frac{(\mathbf{g} \mathbf{r})(\mathbf{P} \mathbf{p}) + (\mathbf{P} \mathbf{r})(\mathbf{g} \mathbf{p}) - i \hbar \mathbf{g} \mathbf{P}}{(m_e + m_p)c^2}, \quad (24)$$

where $\mathbf{g} = -G \frac{M}{R^3} \tilde{\mathbf{R}}$. In Eqs.(20)-(24), $\tilde{\mathbf{R}}$ and \mathbf{P} stand for coordinate and momentum of a hydrogen atom center of mass, respectively. Whereas, \mathbf{r} and \mathbf{p} stand for reciprocal electron coordinate and momentum in the center of mass coordinate system; $\mu = m_e m_p / (m_e + m_p)$ is the so-called reduced electron mass. Note that $\hat{H}_0(\hat{\mathbf{P}}, \hat{\mathbf{p}}, r)$ represents the Hamiltonian of a hydrogen atom in the absence of a gravitational field. On the other hand, $\hat{H}_1(\hat{\mathbf{P}}, \hat{\mathbf{p}}, \tilde{\mathbf{R}}, r)$ describes couplings of the bare electron and proton masses to the gravitational field (2) as well as couplings of kinetic and potential energies to the field. The Hamiltonians $\hat{H}_2(\hat{\mathbf{p}}, \mathbf{r})$ and $\hat{H}_3(\hat{\mathbf{P}}, \hat{\mathbf{p}}, \tilde{\mathbf{R}}, r)$ contain tidal effects.

3.2. Derivation of the Hamiltonian (7),(8)

Let us strictly derive the Hamiltonian (7),(8), which has been semi-quantitatively derived in Section 2, from the more general Hamiltonian (20)-(24). First of all, for simplicity we use the approximation, where $m_p \gg m_e$, and, thus, $\mu = m_e$, which allow to consider proton as a heavy classical particle. In addition, in this Subsection, we derive the Hamiltonian of a hydrogen atom, whose center of mass is at rest in the gravitational field (2). Therefore, we can disregard center of mass momentum and center of mass kinetic energy. As a result, in the case under consideration, the first two contributions to the total Hamiltonian (20)-(24) can be rewritten in the following way:

$$\hat{H}_0(\hat{\mathbf{p}}, r) = m_e c^2 + \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} \quad (25)$$

and

$$\hat{H}_1(\hat{\mathbf{p}}, r) = \left\{ m_e c^2 + \left[3 \frac{\hat{\mathbf{p}}^2}{2m_e} - 2 \frac{e^2}{r} \right] \right\} \left(\frac{\phi}{c^2} \right), \quad (26)$$

where we place the center of mass of the atom at point $\tilde{\mathbf{R}} = 0$. Let us consider the first tidal contribution (23) to the total Hamiltonian (20). Note that $|\mathbf{g}| \simeq |\phi|/R_0$, where R_0 is the Earth's radius. In addition, in a hydrogen atom, $|\mathbf{r}| \sim \hbar/|\mathbf{p}| \sim r_B$ and $\mathbf{p}^2/(2m_e) \sim e^2/r_B$. These values allow us to estimate the Hamiltonian (23) as $H_2 \sim (r_B/R_0)(\phi/c^2)(e^2/r_B) \sim 10^{-17}(\phi/c^2)(e^2/r_B)$, which is 10^{-17} smaller than $H_1 \sim (\phi/c^2)(e^2/r_B)$ and 10^{-8} smaller than the second correction with respect to the small parameter $|\phi|/c^2$, which is omitted. Thus, we can disregard the contribution (23) to the total Hamiltonian (20)-(24). Moreover, we pay attention that the second tidal contribution (24) to the total Hamiltonian is exactly zero in the case, where $\mathbf{P} = 0$, considered in this Subsection. At this point, we can conclude that the Hamiltonian (25),(26), derived above, exactly coincides with that, semi-quantitatively derived by us in Refs.[4–8] [see Eqs.(7),(8)].

3.3. Derivation of the Hamiltonian to Describe the Suggested Experiment [4–8]

In this Subsection, we derive the Hamiltonian for the case, where proton is fixed in a inertial coordinate system, moving from the Earth with a constant velocity $u \ll c$. We relate such inertial system to a spacecraft in accordance with the experiment, suggested in Refs. [4–8] and described in the next Section. As in the previous Subsection, we use here the relationship $m_p \gg m_e$, which allows us to consider proton as a classical particle as well as to disregard its kinetic energy. In the case under consideration, the first two contributions to the total Hamiltonian (20) in the inertial system, related to the spacecraft, can be written as

$$\hat{H}_0(\hat{\mathbf{p}}, r) = m_e c^2 + \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} \quad (27)$$

and

$$\hat{H}_1(\hat{\mathbf{p}}, r) = \left\{ m_e c^2 + \left[3 \frac{\hat{\mathbf{p}}^2}{2m_e} - 2 \frac{e^2}{r} \right] \right\} \left[\frac{\phi(R_0 + ut)}{c^2} \right]. \quad (28)$$

Note that the contribution (23) to the total Hamiltonian (20)-(24) is estimated in the same way, as in the previous Subsection, and, thus, can be disregarded. As to the contribution (24), it is non-zero in our case and has to be estimated separately. We consider a realistic situation, where our spacecraft moves with constant velocity $u \ll \alpha c$, where α is the fine structure constant and, thus, αc is a typical value of electron velocity in the Bohr's model of a hydrogen atom. In this case, the value of the Hamiltonian (24) can be estimated as $(r_B/R_0)(m_p/m_e)(\phi/c^2) \sim 10^{-14}(\phi/c^2)$ and, therefore, can be disregarded. It is important that our suggestion for the experiment is to observe photons emitted by the atoms, excited in the gravitational field, therefore, it is enough to keep in the Hamiltonian (27),(28) only the virial term, which gives non-zero transitions between different energy levels:

$$\hat{H}_0(\hat{\mathbf{p}}, r) = m_e c^2 + \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} \quad (29)$$

and

$$\hat{H}_1(\hat{\mathbf{p}}, r) = \left(2 \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} \right) \left[\frac{\phi(R_0 + ut)}{c^2} \right]. \quad (30)$$

Here, we pay attention that the Hamiltonian (29),(30) exactly coincides with that, semi-quantitatively introduced in Refs.[4–6] and derived from the Lagrangian [9] in Refs.[6, 8].

4. SUGGESTED IDEALIZED EXPERIMENT [4–8]

Below, we discuss an idealized experiment to observe the breakdown of the equivalence between passive gravitational mass and energy, which is a consequence of the derived Hamiltonians (29),(30). We assume that we have a macroscopic ensemble of hydrogen atoms with each of them being in a ground state at initial moment of time, $t = 0$. All these atoms are supposed to be placed in a small spacecraft and all protons are supposed to have fixed positions in the spacecraft. Then, we suggest that the spacecraft is located at distance R^* from the Earth's center and is at rest for some time at $t < 0$ to prepare the following equilibrium ground state electron wave function in the atoms [8]:

$$\tilde{\Psi}_1(r, t) = \left(1 - \frac{\phi^*}{c^2} \right)^{3/2} \Psi_1 \left[\left(1 - \frac{\phi^*}{c^2} \right) r \right] \exp \left[-im_e c^2 \left(1 + \frac{\phi^*}{c^2} \right) \frac{t}{\hbar} \right] \exp \left[-iE_1 \left(1 + \frac{\phi^*}{c^2} \right) \frac{t}{\hbar} \right], \quad (31)$$

where $\phi^* = \phi(R^*)$. It is important that, in the idealized experiment, we consider non-interacting hydrogen atoms, therefore, we can first study behavior of a single atom. We recall that proton position is fixed in the aircraft, which first accelerates for a short time and then moves long enough time with constant velocity $u \ll \alpha c$, which is directed from the center of the Earth. The derived above Hamiltonian (29),(30) does not allow us to consider a short period of the spacecraft acceleration and we disregard it. Instead, we concentrate on a consideration of the main part of the spacecraft trajectory, where it moves with constant velocity, which can be studied by means of the Hamiltonian (29),(30). Note that, on the latter part of the spacecraft trajectory, electrons are excited by gravitational potential

due to its change in time in Eq.(30). Therefore, on the main part of the trajectory, we can expect the following electron wave function [8]:

$$\tilde{\Psi}(r, t) = \left(1 - \frac{\phi^*}{c^2}\right)^{3/2} \sum_{n=1}^{\infty} \tilde{a}_n(t) \Psi_n \left[\left(1 - \frac{\phi^*}{c^2}\right) r \right] \exp \left[-im_e c^2 \left(1 + \frac{\phi^*}{c^2}\right) \frac{t}{\hbar} \right] \exp \left[-iE_n \left(1 + \frac{\phi^*}{c^2}\right) \frac{t}{\hbar} \right], \quad (32)$$

where the perturbation (30) for a hydrogen atom can be expressed as

$$\hat{U}(\mathbf{r}, t) = \frac{\phi(R^* + ut) - \phi(R^*)}{c^2} \hat{V}(\hat{\mathbf{p}}, r), \quad \hat{V}(\hat{\mathbf{p}}, r) = \left(2 \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r}\right). \quad (33)$$

We point out that in the spacecraft, which moves with constant velocity, each electron experiences gravitational force action, $\mathbf{F} = m_e \mathbf{g}$. It is important that, as possible to show, such force just slightly changes electron energy levels in a hydrogen atom and does not produce electron excitations. Here, we apply the standard time-dependent quantum mechanical perturbation theory to calculate electron amplitudes $\tilde{a}_n(t)$ in Eq.(32) for $n \neq 1$,

$$\tilde{a}_n(t) = -\frac{V_{1,n}}{\hbar\omega_{n,1}c^2} \left\{ [\phi(R^* + ut) - \phi(R^*)] \exp(i\omega_{n,1}t) - \frac{u}{i\omega_{n,1}} \int_0^t \frac{d\phi(R^* + ut)}{dR^*} d[\exp(i\omega_{n,1}t)] \right\}, \quad (34)$$

where

$$\omega_{n,1} = \frac{E_n - E_1}{\hbar} \quad (35)$$

and $V_{1,n}$ is given by Eq.(17). We pay attention that, under conditions of the suggested experiment, the following inequality is obviously valid:

$$u \ll \omega_{n,1} R \sim \alpha c \frac{R}{r_B} \sim 10^{13} c. \quad (36)$$

It is easy to see that Eq.(36) means that the second term in Eq.(34) is much less than the first one and, thus, can be omitted below:

$$\tilde{a}_n(t) = -\frac{V_{1,n}}{\hbar\omega_{n,1}c^2} [\phi(R^* + ut) - \phi(R^*)] \exp(i\omega_{n,1}t), \quad n \neq 1. \quad (37)$$

Note that, as directly follows from Eq.(37), there are exist non-zero probabilities to find electron on the levels nS with $n \neq 1$ in Eq.(18) due to electron excitations during the experiment. If the excited levels of the atom were strictly stationary, then the corresponding probabilities would be:

$$\tilde{P}_n(t) = |\tilde{a}_n(t)|^2 = \left(\frac{V_{1,n}}{\hbar\omega_{n,1}c^2} \right)^2 \frac{[\phi(R^* + ut) - \phi(R^*)]^2}{c^4} = \left(\frac{V_{1,n}}{\hbar\omega_{n,1}c^2} \right)^2 \frac{[\phi(R') - \phi(R^*)]^2}{c^4}, \quad n \neq 1, \quad (38)$$

where $R' = R^* + ut$. As well known, in fact, the excited levels of any atom are quasi-stationary ones and, thus, they spontaneously decay with time. In this situation, it is necessary to measure the spontaneous emission of radiation, corresponding to $n \neq 1$ in Eq.(18), which directly manifests the breakdown of the Einstein's equation for passive gravitational mass and energy. [Here, we point out that the dipole matrix elements for optical transitions $nS \rightarrow 1S$ are zero. Under such conditions, we expect dipole transitions, such as $3S \rightarrow 2P$ and $2P \rightarrow 1S$, as well as quadrupole transitions, such as $2S \rightarrow 1S$, etc. Let us estimate the probabilities (38). It is easy to see that, by using spacecraft, we can reach the condition $|\phi(R')| \ll |\phi(R^*)|$. In this case Eq.(38) coincides with Eq.(19), which reflects their common physical meaning, and can be written as

$$\tilde{P}_n = \left(\frac{V_{1,n}}{E_n - E_1} \right)^2 \frac{\phi^2(R^*)}{c^4} \simeq 0.49 \times 10^{-18} \left(\frac{V_{n,1}}{E_n - E_1} \right)^2. \quad (39)$$

Note that, in Eq.(39), we use the following values of the Earth's parameters: $R_0 \simeq 6.36 \times 10^6 m$ and $M \simeq 6 \times 10^{24} kg$. It is important that probabilities (39) are of the second order of magnitude with respect to the small parameter, $|\phi/c^2| \ll 1$, and, thus, small, $P_n \sim 10^{-18}$. On the hand, it is not too small and the corresponding number of the excited electrons in a microscopic ensemble of the atoms can be very large. Indeed, $V_{n,1}/(E_n - E_1) \sim 1$ and the

Avogadro number is $N_A = 6 \times 10^{23}$. Therefore, the number of excited electrons for 1000 moles of the atoms is estimated as

$$N_{n,1} = 2.95 \times 10^8 \left(\frac{V_{n,1}}{E_n - E_1} \right)^2, \quad N_{2,1} = 0.9 \times 10^8, \quad (40)$$

where $N_{n,1}$ determines the number of electrons excited from energy level $1S$ to energy level nS . We hope that the corresponding photons, for such large amount of the excited atoms, can be experimentally detected.

5. DISCUSSION

In this Section, we discuss in brief how to carry out a real experiment in space to detect those seldom events, where the equivalence between passive gravitational mass and energy is broken. We recall that all our calculations in this article have been done for an atomic hydrogen in the approximation of non-interacting atoms. In addition, we have suggested that all protons are fixed in the spacecraft inertial coordinate system. The best realization of these ideal conditions seems to be fulfilled in solid Helium-4, where He atoms are connected by van der Waals' forces. On the other hand, the suggested phenomenon is very general and can be observed for any types of excitations: in solids, in nuclei [14], etc. For instance, we suggest to measure conductivity in semiconductors to detect the excited electrons.

6. CONCLUSIONS

To conclude, we point out that we have suggested to detect photons, emitted by excited electrons in a macroscopic ensemble of the atoms, provided that the atoms are moved in the Earth's gravitational field with constant velocity, $u \ll \alpha c$, by a spacecraft. Our proposal is very different from proposals, discussed before, where small corrections to electron energy levels are calculated for a hydrogen atom, supported in a gravitational field [13], or for a free falling atom [15, 16]. Account of the above mentioned small corrections to energy levels cannot change significantly the number of electrons, excited due to spacecraft motion in an gravitational field (2), calculated in the paper. It is also very important that the suggested experiment is not a "free fall" experiment. For free falling atoms, the effects, discussed by us (in this paper and in Refs. [4–8]), presumably disappear since free falling atoms feel only the second derivative of a gravitational potential. Therefore, we think that comparison of our effects with "free fall" astronomical data, performed in Ref. [14], is not appropriate. In particular, in our opinion, the conclusion of Ref. [14] that our theoretical results contradict to the existing experimental astronomical data is incorrect.

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